

Investing in speculative industries

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1 Introduction

The interest for investigating speculative markets came about from a conversation I had with a close friend in December of 2019. She had made a substantial profit from investing early in marijuana - related companies, thinking that speculation might result in rapid increases in price. Her speculation proved to be correct and she made a profit, at least at first. She kept holding her shares and eventually got to a position where the stock prices plummeted back down, leaving her without the gains she could have made if she sold earlier.

Her analysis of the situation was that speculation drove the prices up, but the stock values did not represent the actual value of the company. Therefore, upon legalization, the prices fell as a consequence of the discrepancy between actual company worth and what was speculated beforehand. Her conclusion was that she should have sold just before legalization of the product.

My argument was that the same strategy that made her buy the stocks in the first place was the same strategy which caused her to not sell then stocks right before legalization, since for all she knew the prices could have gone the other way upon legalization. This got me thinking about how one can form a consistent strategy for investing in speculative markets like this one. Simply claiming that one has learned a lesson from the preceding story is in my opinion simply a way to justify the missed opportunity to capitalize on the opportunity. So this begs the question, how should it have been done?

2 Idea

My idea of a consistent strategy would be to hold a fixed portion of your capital in speculative industries, and re-normalize the proportions on a regular basis. In this way one would expect to capitalize on some of the upside, whilst not suffering as bad if things turn sour. Note that by re-normalizing under a rising period looks sub-optimal since you remove capital from a rapidly growing industry. Yet this might be your best option in light of the fact that you don't

know when the winds will change. To test whether or not this strategy has any potential we make use of a simple model.

3 Model

Assume we have n number of speculative stocks we can invest in, who all follow geometric Brownian motions of the form :

$$dS_t^{(i)} = S_t^{(i)}(X^{(i)}dt + \sigma dW_t^{(i)})$$

Where $W_t^{(i)}$, $i = 1, \dots, n$ are independent Brownian motions and the random variable $X^{(i)} \sim N(\bar{\mu}, \bar{\sigma})$. I.e. we assume that all speculative stocks have drifts that are independent realization of the same normal distribution. We assume that the mean $\bar{\mu}$ and variance $\bar{\sigma}^2$ of this distribution are known. The questions we ask is what returns we can expect if we consider an equally weighted portfolio of these speculative assets.

The expected return of this portfolio is:

$$\begin{aligned} R_t &= \mathbb{E} \left[\frac{S_t^{(1)} + \dots + S_t^{(n)}}{n} \right] = \mathbb{E} [S_t^{(1)}] \\ &= \mathbb{E} [\mathbb{E}[S_t^{(1)} | X^{(1)} = x]] = \mathbb{E} \left[e^{(x - \frac{\sigma^2}{2})t} \cdot \mathbb{E}[e^{\sigma \sqrt{t} W_t^{(1)}}] \right] \\ &= \mathbb{E} \left[e^{(x - \frac{\sigma^2}{2})t} \cdot e^{\frac{\sigma^2 t}{2}} \right] = \mathbb{E} [e^{xt}] = \mathbb{E} [Y] \end{aligned}$$

Since $X^{(1)} \sim N(\bar{\mu}, \bar{\sigma})$ we get that Y is log-normal with parameters $\bar{\mu}t$ and $\bar{\sigma}^2 t^2$, i.e. $X^{(1)} \sim \text{LogNormal}(\bar{\mu}t, \bar{\sigma}^2 t^2)$. Therefore we get that

$$R_t = \mathbb{E} [Y] = \exp \left(\bar{\mu}t + \frac{\bar{\sigma}^2 t^2}{2} \right)$$

Note that the higher the volatility $\bar{\sigma}$ of the underlying drift-distribution, the higher the expected return. The variance of the portfolio return is given by:

$$[\exp(\bar{\sigma}^2 t^2) - 1] \exp(2\bar{\mu}t + \bar{\sigma}^2 t^2)$$

Note that a higher volatility $\bar{\sigma}$ of the underlying drift-distribution is also associated with a higher overall portfolio variance.

4 Conclusions

From the preceding discussion it's clear that for large enough $\bar{\sigma}$ the expected return of the portfolio will outperform other types of investments, such as index funds. This is a compelling reason to invest in speculative assets. Acquiring good estimates of the parameters is however a hard task and it is therefore unclear how large of a proportion, if any at all, an investor should dedicate these assets.